

Reconceptualized Perspective to Mathematics: Effects on Students' Mathematics Achievement and Learning Experiences

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Abstract: The main objective of the study is to contribute a reconceptualized perspective of mathematics which is a harmonic blend of the opposing philosophical views of mathematics – absolutist and constructivist. The study also aimed to contribute worthwhile classroom implications of this alternative perspective in mathematics and mathematics teaching. The study used the quasi-experimental three-group pretest-posttest research design to determine the effects of the teaching approaches: Strict Absolutist, Strict Constructivist and Reconceptualized on students' mathematics learning experiences and achievement. The study involved 112 college freshmen taking up BS Mathematics program. Mathematics Learning Experiences Inventory and a mathematics achievement test were developed and validated for the purpose of this study. The results of the study revealed that the Reconceptualized approach to mathematics teaching can better improve students' mathematics learning experiences and achievement than the other two teaching approaches: Strict Absolutist and Strict Constructivist. Based on the results of this study, it is being proposed that providing students with rich learning opportunities that foster better understanding of the nature of mathematics together with its history and vast application can better escalate students' learning performance and enrich their experiences in mathematics.

Keywords: Mathematics, Education, Reconceptualized Perspective, Philosophical Views

Introduction

If one would trace the history of mathematics, one can discover the numerous attempts of educators and philosophers in mathematics to reform the long-standing view of mathematics - absolutist perspective. However, the attempts made has been confined to ideas in such rigid convention and offer the other end of the stick that views mathematics as subjective, relative and fallible. Those who advocate the strict absolutist perspective to mathematics teaching (traditionalist) view mathematics as a given and ready-made truth to be passed on to students by employing the prevalent lecture-discussion method. On the other hand, those who advocate a more child-centered and applied approach to mathematics teaching (strict constructivist) view mathematics as generated in the minds of the thinking subject. Hence, students in a strict constructivist classroom are encouraged to construct their own learning which is an exact opposing idea to that of the traditionalists. And because of these opposing views of mathematics, and consequently opposing approaches to mathematics teaching, the educators and practitioners have to choose which camp to take.

The radical strict constructivism perspective to mathematics argues that knowledge, no matter how much it is defined, is something that is made and constructed by the thinking individual in relation to his or her own experience (Von Glasersfeld, 1995). The most notable

contribution of constructivism is that it offered ways in which we struggle to come to know and do mathematics, however the worst that it has done as Stemhagen (2004) described it, constructivism denied mathematics its stability and universality. This dualistic clash had caused much damage and deterred the growth of mathematics and mathematics education. One has to realize that if he picks up one end of the stick, he also in one way or another had picked up the other end. Hence, this study is a proposition for a complementing blend of these perspectives to engender harmonic educational practice.

In response to the gap and issues discussed above, firstly, this study aims to contribute a reconceptualized perspective of mathematics that regards the unique universality of mathematics and recognizes it as a developing and evolving discipline. To the knowledge of the researcher, this reconceptualized perspective to mathematics have not been written and studied as reflected in the literature. This perspective suggests that mathematics is a stable body of knowledge that has the ability to grow and expand in the face of new discovered or created information. The researcher strongly argues that in the emergence of new knowledge, previous mathematical truths are not discarded. Instead, the specificity of the domain validity of the existing concept is expanded to cater a new realm. This reconceptualized perspective is an alternative view of mathematics that recognizes both its absolute stability vis-à-vis its adaptability. Moreover, this reconceptualized perspective is hoped to offer a potential softening of the entrenched divide between the strict absolutist and strict constructivist views of mathematics. As a consequence of this reconceptualized perspective to mathematics, secondly the study aims to contribute pedagogical implications of such reconceptualization. This study aims to establish that for mathematics education to prosper it can be useful to think of it as partly found and partly made. The reconceptualized approach to mathematics education suggested in this study highlight the significance of engaging students to worthwhile learning opportunities that present the developmental nature of mathematics and encourage students to become creative in mathematics.

This reconceptualized approach to mathematics teaching further suggests that the matching of the mathematics given to student and the student's prior knowledge is an integral proactive step in bridging the gap in mathematics learning. Lev Vygotsky, a Russian psychologist, proved that individuals learn best in accordance with their readiness (Allan & Tomlinson, 2000). That is, the difficulty of skills taught should be slightly in advance of the child's current level of mastery. This is grounded in Vygotsky's work and the zone of proximal development (ZPD), the range at which learning takes place. Psychologists tell us that a student learns only when a task is a little too hard for the student. When a student can do work with little effort the student is not learning, but rather rehearsing. When a student finds a task beyond his reach, frustration, not learning, is the result. Only when a task is a bit beyond the student's comfort level, and the student finds a support system to bridge the gap, does learning occur. This theoretical influence provides a concrete foundation for the reconceptualized approach being suggested in this paper. In this approach, the inevitable "some" way of transmission that takes place as certain mathematical concepts and truths are defined and elaborated to students is being recognized. In this reconceptualized approach individualized mathematical solutions and methods are encouraged. It suggests that teachers may present mathematical concepts and allow students to

practice mathematical skills in absolutist back-to-basic approach which can be done through demonstration, repetition and drill. This is done until such time that students gain understanding and mastery of the mathematical concepts and skills. However, it is strongly suggested that after attaining mastery of mathematical skills and concepts, students are given meaningful bona fide problem-solving activities that are within the interest and passion of the students. In this way, students realize that mathematics is something that they need, can do and create. This is hoped to foster deeper understanding of the concepts, inculcate appreciation of the use and engender mathematics discovery or creation.

Furthermore, this reconceptualized approach to mathematics pedagogy suggests that teachers incorporate rich historical account of mathematical concepts to each lesson whenever possible so as to provide students the necessary background, understanding and appreciation of the discipline. This will provide students better understanding of the nature and purpose of mathematics. These meaningful additions are believed to foster increased mathematics interest and curiosity which in turn can result to increased mathematics engagement and learning.

This study delved more deeply into the possible pedagogical implications of this alternative approach by comparing it to two other most prevalent pedagogical approaches, strict absolutist or the traditional lecture-discussion method and the unconventional strict constructivist approach.

The following table presents the comprehensive comparisons of the three mathematical perspectives and philosophies.

Table 1: Comparison of Strict Absolutist, Strict Constructivist and Reconceptualized Perspectives

Absolutist	Constructivist	Reconceptualized
Mathematical knowledge is absolute truth, objective, logical, permanent and incorrigible.	Mathematical knowledge is subjective, tentative, relative and fallible.	Mathematical knowledge is stable, objective and capable of growing and evolving - the specificity of the domain validity of the concept is expanded to cater to new mathematical knowledge.
Mathematical reality lies outside us and all that man can do is to discover it.	Mathematical reality is in the head of the thinking subject and that all he can do is to construct what he knows on the basis of his experience.	Mathematical reality is a mental activity where concrete and absolute truths is accessed and understood through the senses to make meaningful relations.
Mathematics has no necessary relation to the world we live in.	Mathematics helps us cope in the world we live in.	Mathematics is a functional tool to better understand the world.
Mathematics exists outside of human influence.	Mathematics is a human-meaning making venture.	Mathematics is produced by human activity.
Mathematics is a given and ready-made truths to be passed on to students.	Students construct their own mathematics understanding. They are the generators of knowledge.	Acknowledges the objective formal mathematics which are used for mathematical intuition and thinking processes.

Mathematics classroom practice is one-size-fits-all.	Students accommodates and adjust their knowledge structures to the given mathematical problem.	Values appropriate matching of content and the individual's capacity for a meaningful interaction and construction of understandings.
Mathematical solutions are constructed based from the teacher's demonstration.	Exploration and discovery of mathematical solutions.	Individualized mathematical solutions and methods are encouraged.
Nature of mathematics understanding and history of mathematics are hardly imparted.	Implied exposing to the nature of mathematics.	Explicit teaching of the nature of mathematics and rich incorporation of mathematics historical account.

Literature Review

In terms of teaching, the “melding of the absolutist and fallibilist perspectives, or philosophical movement within the arena of mathematics perspectives, may be essential for authentic learning to occur”, (Davison & Mitchelle, 2008) thus ensuring that the combination of the two perspectives yields equitable form of utilizing philosophy and technical knowledge in teaching. The contextualized perspective to mathematics in this study, recognizes that mathematics is a stable body of knowledge that is certain, objective, and systematic, but can change, grow, and expand, engendered by continuous utilization and interaction of humans, to its realm.

The researcher used several theories of mathematics philosophy and mathematics education to establish a reasonable acceptance of the alternative approach. One of these is Kitcher's theory. Kitcher argued that the “originators of mathematics made their mathematical observations in the midst of trying to solve practical problems” (1976). His ideas of empiricism through the notion of perception as a critical component of the origins of mathematics puts into perspective the importance of mathematics history as an integral element to understand the concepts of mathematics as a subject. In a similar line of thought, Imre Lakatos proposed quasi-empiricism as a theory of mathematics that rejected absolute certainty as a goal and acknowledged the historical, hypothetico-deductive nature of mathematics (Ernest, 1990, p.34). He called his way of thinking of mathematics as “quasi-empiricism” to show that albeit science and mathematics are similar in method, both differ in content. And that unlike science – wherein the truths are verified not by physical objects or observations – mathematics' truths are verified by mathematical ideas. According to Stemhagen (2004), Kant agreed that the mind's categories help to organize information from the senses in everyone's mind. However, he emphasized that the structures are themselves universal and absolute. This argument undergirds the propositions of this study that mathematics can be better viewed as partly found and partly made.

It is argued in this study that to foster better mathematics student learning, the nature of mathematics needs to be understood and embodied by mathematics teachers. Dewey argued that the demand for a teacher is two-fold. The teacher must have a thorough knowledge of the discipline and an awareness of those common experiences an individual has that can be utilized to lead them toward understanding (Stemhagen, 2004). Vygostky's (1986) Zone of Proximal

Development (ZPD) can be a guiding theory in tailoring classroom activities to students' prior knowledge to bridge the gap between what is given and what students already know. This method of providing students the opportunity to apply their acquired knowledge will also impart philosophical ideas of mathematics: That mathematics knowledge with its absolute and objective nature can grow and expand. Ernest (1990) realized that the issue of the relationship between philosophies of mathematics and mathematics education is the central problem of mathematics education. The nature of mathematics is central to the philosophy of mathematics education. This also has significant practical outcomes and implications for the teaching and consequently learning of mathematics. However, it has been observed in the past that when the nature of mathematics is realized in practice, it will create conflicts that result in the siding to either of the two camps of absolutism or relativism. Absolutism, sometimes also called infallibilism or apriorism, is a family of related approaches and thinking constituting the predominant philosophy of mathematics (Ernest, 1991). In this view of mathematics, absolute, certain, and unchallengeable truth is rendered to the nature of mathematical knowledge. The most general argument of non-absolutist philosophers is their strong opposition to the absolutist perspectives. Hersh (1979) explains that "it is reasonable to propose a new task for mathematical philosophy: not to seek indubitable truth but to give an account of mathematical knowledge as it is – fallible, corrigible, tentative and evolving, as is every other kind of human knowledge" (p. 43). Glasersfeld (1983) offered the idea that knowledge is the result of a learner's activity rather than of the passive reception of information or instruction. He further elucidates how cognition helps: "us to cope in the world of our experience, rather than the traditional goal of furnishing an 'objective' representation of the world as it might 'exist' apart from us and our experience".

Methodology and Methods

The quasi-experimental method of research particularly the pretest-posttest research design was used for the conduct of the comparative analysis on the influence of Reconceptualized perspective, strictly absolutist and strictly constructivist approaches on students' performance and learning experiences.

Where			
O ₁ , O ₂ , O ₃ – pretests			
O ₁ ', O ₂ ', O ₃ ' – posttests			
X ₁ – teaching method conventional ap	O ₁	X ₁	O ₁ '

X ₂ – teaching method approach)	O ₂	X ₂	O ₂ '

X ₃ – teaching method (Reconceptualiz	O ₃	X ₃	O ₃ '

Fig. 1 Research Design

Since classes are intact groups and randomization of sample is difficult to conduct, consisting of three groups, the design gives pretest and posttest to each group but only two gets the treatment, which in this case, is the strictly constructivist and reconceptualized approach. The controlled group was the strictly absolutist approach or the traditional-lecture discussion method of teaching. The broken line between the groups indicates randomization of participants between groups is not done. Purposive sampling was used to determine the three intact classes

which will be part of the study. The study involved 112 college freshmen students who were distributed in the three classes.

Two (2) research instruments were developed and validated for the purpose of this study. These research instruments were used to gather the necessary data and information that shed light to the objectives of this study.

Mathematics Learning Experience Inventory (MLEI)

A Mathematics Learning Experience Inventory was constructed and validated for the purpose of this study. This learning experience inventory aims to delve deeply into the varied learning experiences of the students to further provide information and support students' inputs in their learning journals and reflection exit cards. The questionnaire comprises thirty (30) items that delve deeply into students' opinion on how they view and appreciate their learning activities in the three learning approach groups namely: strict absolutist, strict constructivist, and absolutist-constructivist approach. The rating scale of the inventory was interpreted as follows: 6 – Strongly Agree, 5 – Moderately Agree, 4 – Slightly Agree, 3 – Slightly Disagree, 2 – Moderately Disagree and 1 – Strongly Disagree. The development and validation process involves the following stages: planning the test, face validation, first trial run, item analysis, final run and validity tests.

The questionnaire was administered to 120 college students. Table 2 presents the descriptive statistics of the instrument.

The final draft of the questionnaire was then subjected to confirmatory factor analysis (CFA) to test if the factor structure is true. Table 3 shows that all items demonstrated high factor loadings ranging from .432 to .735 values. This factor loadings indicate high correlation of each item with the component factor structure of the questionnaire. This results further indicate that the scale has appropriate factor structure and is valid in measuring the constructs under study.

Table 2: Descriptive Statistics of Final Form of MLEI

Construct	Question	Mean	Standard Deviation
Learning Appreciation	1	4.63	.76
	2	4.37	.98
	3	4.61	.70
	4	3.31	1.07
	5	4.31	.77
	6	5.08	.79
	7	4.27	1.30
	8	4.83	.93
	9	4.76	.86
	10	4.93	.69
	11	4.73	.74
	12	4.98	.97
	13	5.14	.71
	14	4.68	.94
	15	4.53	.86
Mathematics Learning Strategy	1	4.92	1.04
	2	4.44	1.06
	3	2.86	1.20
	4	3.78	1.05

5	4.75	1.12
6	4.54	1.06
7	4.39	1.05
8	4.95	1.01
9	4.22	1.25
10	3.15	1.35
11	4.17	1.11
12	5.00	.97
13	4.51	1.09
14	2.51	1.18
15	4.64	.96

Table 3: Factor Analysis of Final Draft of MLEI

Question	Construct	Item	Factor Loading
1	Learning Appreciation	I find most of what I learned in our math course very interesting and useful.	.727
2		I enjoy getting involved in math activities where I can work with my classmates.	.468
3		I appreciate the way our math lessons and activities are designed.	.445
4		I tend to get bored and lose interest in the middle of our math class.	.432
5		Our math class gives me more confidence in learning and doing math.	.621
6		I like that our teacher is very enthusiastic in discussing our math lessons and activities.	.678
7		I don't think I learned so much from our math course.	.566
8		I like that our teacher gives us the opportunity to solve math problems in our own way.	.471
9		I like that there is a motivating part in our math class where we are encourage to think and learn on our own.	.614
10		I like that there are opportunities in our math class where we work with co-learners.	.492
11		The performance feedback of my teacher and classmates help me improve my learning and understanding of mathematics.	.636
12		I like that our teacher discusses specific math procedures but allows us to solve problems using different methods.	.527
13		I learned recent and interesting things about math which I did not know before.	.645
14		I like that our math class has many unique and interesting activities.	.678
15		Through our math lessons and activities, I gained more confidence in dealing with math.	.572
1	Mathematics Learning Strategies	Every time I solve math problems, I go over my answer again to see if it make sense.	.538

2	I try to redo sample math problems and see if I can solve them using other methods.	.519
3	When I study math lessons, I simply read and memorize the concepts and procedures.	.587
4	Every time I encounter a new math concept, I try to research more about.	.601
5	If I don't understand a math lesson well, I will try a different approach like research and watch tutorial videos about it.	.593
6	Every time I don't solve math problems, I often have trouble making sense of the concept.	.563
7	In studying for mathematics, I provide sufficient time.	.735
8	I try to look for simpler ways of solving a math problem.	.507
9	To understand better math concepts, I try to relate it to real life situations and apply it into practical use.	.645
10	When I can't solve a math problem after a few minutes, I tend to give up on it.	.563
11	I solve mathematics problems over and over again to learn it.	.694
12	It helps me understand math more if I can see and visualize the concepts.	.527
13	I try to draw or make representations so I can easily understand math problems.	.553
14	I solve math problems which I think I can and skip those that I find very difficult.	.563
15	Reading more about a math concept helps me understand it better.	.550

The Cronbach's alpha coefficient of each subscale and the overall instrument was obtained to assess the construct validity. It was found out that all the items used in this instrument has excellent internal consistency as revealed by the alpha coefficients shown in Table 4, thereby exceeding the recommended reliability alpha estimates of $\alpha=.70$.

Table 4: Cronbach's alpha Reliability Coefficients of MLEI

Construct	No. of Items	Alpha
Learning Appreciation	15	.848
Mathematics Learning Strategies	15	.813

The validation process done ensured that the Mathematics Learning Experiences Inventory is a good and valid measure of the constructs included in the instrument.

Mathematics Achievement Test

An achievement test in mathematics was developed for the purpose of this study. This test aims to measure the level of knowledge students have about the following topics Propositions, Tautologies and Logical Equivalence, Arguments and Proofs, Sets and Operations, Set Identities

and Laws of Algebra of Sets, Divisibility and Euclidean Algorithm. A Table of Specification (TOS) was prepared to evenly distribute the content of the items across Bloom's Taxonomy of learning. This achievement test in mathematics is a multiple-choice test that has four options in each item.

The first draft of the mathematics achievement test was pilot tested to 120 (one hundred twenty) college students. The data from this trial run was analyzed using Item-Response Theory (IRT) or sometimes also called as Latent Response Theory. This analysis model is used to evaluate the relationship if there is any between a latent trait which in this case is knowledge in mathematics and the items in the test. This can also be used to measure the relationship between the level of mathematics knowledge (latent trait) and the instrument as a whole. IRT model particularly the 3PL (3 Parameters Logistic Model) was used to analyze each item in the mathematics achievement test developed for this study. This IRT model type predicts the probability of a successful answer using three parameters – item difficulty, item discrimination and the pseudo chance (guessing). The idea in this instrument analysis model is that the position of an individual on the latent construct continuum vary according to their abilities. Hence, an item in a test must be sensitive enough to position the respondent within the suggested unobservable continuum. Item difficulty is the parameter that determines the manner of which the item behaves along the ability scale. Item discrimination is the parameter that determines the rate at which the probability of endorsing a correct answer change based on respondent's ability levels. The idea is to include items in the test that has high discrimination value to ensure precise measure of the latent trait. This is done to map individuals along the continuum of the latent trait which is knowledge in mathematics. A third parameter, guessing value, is considered in this item analysis to ensure that correct answers are caused by respondent's high ability. When respondents answer the item by guessing, the amount of information provided by that item decreases and it indicates that the respondent's ability is lesser than the item difficulty. Hence, item with low item information function values is discarded. As a result, ten (10) items were omitted from the first draft of the test. These items are also the items which were marked as weak by the mathematics expert evaluators. The summary of the Stata IRT analysis is presented in Table 5 below. The items which have the lowest discrimination index were deleted from the first draft of the test. On the other hand, items that were retained but has negative difficulty index were revised to improve structure and complexity. Moreover, the guess index value of .11 indicates that there is an 11% probability that a respondent can guess the answer correctly. This guess index value reveals a small percentage which makes the questions strong and good items.

The -0.630-correlation coefficient of the achievement test is negative because the grading system in the university have 1.00 as the highest and 5.00 as the lowest. The test reliability of .768 determined using Kuder-Richardson Formula 20 indicates high reliability of the test.

At the start of the semester, the teacher-researcher administered the MLEI and achievement test in mathematics. The researcher taught the three learning groups: strict absolutist, strict constructivist and the reconceptualized approach group. The Student's Mathematics Learning Experiences Inventory (MLEI) and achievement test were administered again which served as the posttest scores of the students.

Table 5: Item Response Theory - Three Parameter Logistic (3PL) Model

Question	Discrimination	Difficulty
10	16.906	-.950
7	9.385	2.129
6	6.693	2.142
17	5.272	.298
3	3.992	.995
22	3.864	1.435
33	2.955	.130
16	1.547	-.250
43	1.509	1.653
2	1.428	.564
38	1.152	-1.674
12	1.034	.091.
34	1.027	-.822
15	.984	.841
5	.899	-.648
40	.870	.263
46	.845	-2.239
36	.838	-4.211
9	.818	1.009
37	.787	-4.428
1	.668	-.496
26	.662	-.369
8	.661	18.160
29	.619	1.471
35	.596	.485
41	.582	1.200
4	.568	-1.775
21	.502	-.629
44	.465	1.462
19	.356	2.213
49	.306	-1.499
48	.193	-.947
13	.097	32.912
39	.095	-1.71
31	.071	138.99
30	.063	-51.53
45	.038	12.54
23	-.050	-7.22
20	-.107	9.67
18	-.178	2.76
27	-.188	4.21
32	-.412	6.22
14	-.506	1.55
11	-.593	-5.45
28	-.899	-2.06
42	-1.824	.45
47	-2.152	-.64
25	-3.540	-2.21
Guess	.1107	

Results and Discussion

Mathematics Learning Experience

The Mathematics Learning Experience posttest mean scores of the students in the three learning groups were analyzed using One-Way ANOVA. This is to determine whether there is a significant difference in the learning experiences of the students who are subjected to the three learning approaches: Strict Absolutist, Strict Constructivist and Reconceptualized. The results of this analyses are presented in the following discussions.

Table 6: Means and Standard Deviation for Students' Mathematics Learning Experiences
Posttest Scores

Learning Groups	<i>n</i>	Mathematics Conception	
		<i>M</i>	<i>SD</i>
Strictly Absolutist	35	4.34	.46
Strictly Constructivist	39	4.42	.45
Reconceptualized	38	5.00	.46

Table 6 shows that the students in the Reconceptualized approach group had more engaging learning experiences ($M = 5.0$, $SD = .46$) compared to the students in the Strictly Absolutist ($M = 4.34$, $SD = .46$) and Strictly Constructivist ($M = 4.42$, $SD = .45$). On the other hand, the mean scores of the Strict Absolutist and Strict Constructivist groups are close indicating that the students in the two approaches had nearly similar learning experiences.

A One-Way Analysis of Variance was performed to determine whether the differences in the posttest mean scores of the students in the three learning groups differ statistically significant. The result of this analysis is presented in the next tables.

Table 7: One-Way ANOVA of Students' Mathematics Learning Experiences Posttest
Scores

Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>p</i>
Between Groups	2	9.91	4.96	23.82	.000
Within Groups	109	22.67	.208		
Total	111	32.56			

Table 7 presents that the posttest mean scores of the students from the three approach groups differ statistically significantly, $F = (2, 109) = 23.82$, $p = .000$. This indicates that there are at least two groups that have posttest mean scores that are significantly different in terms of mathematics students' learning experiences. Tukey's HSD post hoc analyses was performed at .05 level of significance. Furthermore, the eta squared ($\eta^2 = 0.30$) was calculated to determine the effect size of the difference of the mean scores. This implies that 30% of the difference among the posttest mean scores of the students in the three learning groups can be accounted for by the type of learning approach given to them. The result of the post hoc multiple comparisons is presented in Table 8.

Table 8: Tukey's HSD Post Hoc Analyses Result of Mathematics Attitude Posttest Scores

(I) Teaching Methodology	(J) Teaching Methodology	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	-.08	.106	.741	-.33	.17
	Reconceptualized	-.67	.107	.000	-.92	-.41
Constructivist	Reconceptualized	-.59	.104	.000	-.83	-.34

*. The mean difference is significant at the 0.05 level.

Table 8 presents that the posttest mean scores of the students in the Reconceptualized approach group is significantly higher than that of the posttest mean scores of the students exposed to Strict Absolutist ($M = .67$) and Strict Constructivist ($M = .59$) at .05 level of significance, $p = .000$ in terms of their learning experiences. This indicates that the students in the Reconceptualized approach had more meaningful learning experiences.

Below are some of the unedited responses of students in their learning journal emphasizing that the Reconceptualized approach provides worthwhile learning experiences which helps students understand the use and benefits of mathematics:

Student C: "I sometimes wonder where can I use mathematics, but because of the activities in our class I realize that math is very useful."

Student G: "I find my learning very interesting and motivating because I understand the origin of mathematics and where to use it."

However, it was found out that the posttest mean scores of the students in both the Strict Absolutist and Strict Constructivist learning groups ($M = .08$) is not statistically significant, $p = .741$ at .05 level of significance in terms of their mathematics learning experiences. This means that the students will have the same mathematics learning experiences when taught using the Strict Absolutist or Strict Constructivist approach. This may be accounted for the extreme treatment used in the two learning approaches. In the Strict Absolutist approach students are provided with the mathematics knowledge and skills using demonstration method. On the other hand, while in the Strict Constructivist approach to mathematics teaching students are encouraged to create or structure their own learning. This only emphasized the need for the right mending of the absolutist and constructivist perspective of mathematics and mathematics teaching. The Reconceptualized perspective in this study suggests that mathematics is better regarded as partly found and partly made. It is the mind that processes, analyzes and construct knowledge as the thinking individual make sense of his experience utilizing the concrete and absolute truths of mathematics. This is well supported by the propositions of Descartes that the mind did take mental activities he strongly argues that it was still an uncovering of preexistent, indubitable type of knowledge.

Mathematics Achievement

One-Way Analysis of Variance was performed to the pretest scores of the students across the three learning groups to assess whether they are comparable before the start of the

experimentation. Table 9 shows the descriptive statistics of Students' Mathematics Achievement pretest score.

Table 9: Means and Standard Deviation of Students' Mathematics Achievement Pretest Scores

Learning Groups	<i>n</i>	Mathematics Achievement	
		<i>M</i>	<i>SD</i>
Strictly Absolutist	35	17.66	4.23
Strictly Constructivist	39	19.54	2.69
Reconceptualized	38	17.58	3.78

It can be observed from Table 9 that the students in the Strict Constructivist group had the highest mathematics achievement pretest scores. This implies that before the implementation of the intervention, the students in the Strict Constructivist group have more knowledge of the mathematics topics discussed during the experimentation.

Also, it can be gleaned from Table 9, that the students in the Reconceptualized learning group had a high mathematics achievement pretest scores with 17.58 (*SD* = 3.78). One-Way ANOVA was conducted to determine whether this difference is statistically significant. Table 10 presents the result of this analysis.

Table 10: One-Way Analysis of Variance of Students' Mathematics Achievement Pretest Scores

Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>p</i>
Between Groups	2	94.02	47.01	3.63	.030
Within Groups	109	1412.84	12.96		
Total	111	1506.86			

Table 10 presents the ANOVA results of the students' mathematics achievement pretest scores of the three learning groups. It can be observed that the ANOVA results implies that there is a significant difference in the pretest mathematics achievement mean scores of the students. This indicated that the three groups are not comparable in terms of the knowledge in mathematics, particularly with the topics included in the experimentation. Thus, careful analysis is needed in analyzing the student's achievement in mathematics. Analysis of Covariance (ANCOVA) was used to determine whether the posttest scores of the students differ statistically significant with the pretest scores used as the covariate.

Table 11: Means and Standard Deviation of Students' Mathematics Achievement Scores

Learning Groups	<i>n</i>	Mathematics Achievement	
		<i>M</i>	<i>SD</i>
Strictly Absolutist	35	26.03	3.85
Strictly Constructivist	39	27.08	3.26
Reconceptualized	38	30.66	2.98

Table 11 shows that the students in Reconceptualized learning group had the highest mathematics achievement mean score ($M = 30.66$, $SD = 2.98$). This implies that after the experimentation done, the students in the Reconceptualized learning group gained better learning and understanding of the mathematics concepts taught. This undergirds the claim that the Reconceptualized approach to teaching mathematics may maximize students' learning and escalate their mathematics performance. This supports the proposition of this alternative view in mathematics teaching those exposing students to the rich history of mathematics, explicitly teaching the nature of mathematics, engaging students to learning activities that is relevant to them, and allowing them to be creative in mathematics can better improve their learning.

The following students' inputs in their learning journal support the claim that the Reconceptualized approach improves students' mathematical thinking and learning:

Student C: "I think the teaching now makes my mind think deeper."

Student S: "The activities given to us are different to the usual, it is very interesting."

Table 12: One-Way Analysis of Covariance of Students' Mathematics Achievement

Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>p</i>	Partial Eta Squared
Corrected Model	3	760.30 ^a	253.43	29.96	.000	.45
	1	1504.4	1504.4	177.8	.000	.62
Intercept		6	6	6		
Pretest Exam	1	322.73	322.73	38.15	.000	.26
Groups	2	519.68	259.84	30.72	.000	.36
Error	108	913.56	8.46			
	112	89258.				
Total		00				
	111	1673.8				
Corrected Total		6				

One-Way Analysis of Covariance was used to analyze if there is a significant difference between the posttest mathematics achievement scores of the students. This is done to account for the similar level of background knowledge of the students prior to the experimentation.

Table 12 shows that teaching methodology used has a significant effect on students' mathematics achievement after controlling students' initial differences in pretest scores, $F = (2,111) = 30.72$, $p = .000$, at .05 level of significance. This is further supported by the partial eta squared of .36 which implies that 36% in the difference in the mathematics achievement scores of the students is explained by the learning methodology implemented.

To determine which among the three groups differ significantly in mathematics Quiz Achievement mean scores, pairwise comparisons analyses were done. The results of this analyses are discussed below.

Table 13: Pairwise Comparisons Analyses Result of Mathematics Conception Posttest Scores

(I) Teaching Methodology	(J) Teaching Methodology	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	-.15	.69	1.00	-1.83	1.54
	Reconceptualized	-4.67	.68	.000	-6.32	-3.01
Constructivist	Reconceptualized	-4.52	.68	.000	-6.17	-2.86

*. The mean difference is significant at the 0.05 level.

It can be observed from Table 13 that the mean scores of the students in the Reconceptualized learning group is higher than that of the other two groups. This implies that the Reconceptualized approach to teaching mathematics can help students maximize their learning.

Conclusions

The Reconceptualized teaching approach can better improve students' mathematics learning and achievement. This implies that utilizing this alternative approach to teaching mathematics can significantly impact students' learning experiences by providing them learning opportunities that expose them to the history and genesis of the mathematics concepts that they are learning, and explicitly teaching them the nature of mathematics. Through this, students will realize that mathematics is partly found and partly made. This also implies that allowing students to be creative in mathematics can foster better learning experiences for students and consequently, better learning.

Ethical Approval

The researcher took the necessary steps to observe research ethics in the conduct of the study by seeking permission from the persons involved in the study. Informed consent was sought from the participants, stating the purpose, risk, and benefits of their participation in the current study and that their participation will be voluntary. The researcher also ensured that the participants' identity and data were treated with the utmost confidentiality.

A. Protection of Human Rights. The participants were kept safe throughout the research process. The researcher made sure that no respondents were exposed to any unnecessary physical or psychological harm during the research process.

B. Risk-Benefit Assessment. The researcher ensured that the respondents were not harmed at any time. Before analyzing the data, the well-being of the respondent was taken into consideration. Data collection was conducted in the safest and most convenient place to ensure quality data gathering.

C. Content, Comprehension and Documentation of the Informed Consent. There were guidelines that were followed to safeguard the welfare of each respondent.

Study Goals. The researcher briefly explained to the respondents the purpose of the study and the importance of their responses. The researcher clearly defined the confidentiality of their participation.

Type of Data. This study collected data using the descriptive survey wherein responses were treated statistically in accordance with research questions on this paper.

Procedures. The researcher read the content of the consent form to all respondents. The consent form contained the signature of the respondents, which served as an agreement of trust.

Nature of Commitment. The researcher allowed the respondents to answer assessment test based on the approved date and time.

Sponsorship. The researcher did not give anything in exchange for the favor requested. The respondents' participation was purely voluntary.

Potential Risks. No significant risk was identified or foreseen in the conduct of the study. The researcher ensured respondents' emotional and physical safety.

Compensation. There was no token or monetary compensation involved in this study. A sincere appreciation and gratitude were given for their voluntary participation.

Confidentiality Pledge. The researcher placed high regard on anonymity and confidentiality. The respondents' identity was not disclosed outside the parameters of the research study.

Right to Withdraw and Withhold Information. Study respondents had the right to withdraw and withhold the data they provided, particularly if they were not comfortable with the confidentiality procedures or if the confidentiality pledge was compromised or if the respondent's privacy was invaded.

D. Privacy and Confidentiality. Study respondents had the right to expect that the data they provided would be kept in strict confidence. The respondent's right to privacy was protected through the various confidentiality procedures. In this study, anonymity and confidentiality were secured.

E. Debriefing, Communications and Referrals. In this study, the participants were not subjected to any psychological discomfort (such as stress, loss of self-esteem, or embarrassment) caused by the research. Therefore, debriefing statements or scripts were not necessary.

F. Vulnerability Assessment. The researcher made sure that the respondents did not belong to the vulnerable population. All the respondents were college freshmen of legal age, free of physical, mental, and emotional illnesses, and could make sound decisions for themselves.

G. Collaborative Study Terms of Reference. This research study was an initiative of the researcher. Permission and agreement were not required for the publishing of this study under intellectual property and publication rights.

Competing interests

The author declares no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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