

Similarity Analysis of the Unsteady Heat and Mass Transfer for The Powell-Eyring Fluid Flow Over Permeable Sheet

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Abstract: This paper focuses on the same phenomenon with Dufour and Soret effects. The system of the Partial Differential Equations represents the behaviour of the flow for different characteristics. Under the symmetry groups of transformations using the order reduction technique, these equations are transformed into the system of ordinary differential equations. These ordinary differential equations are solved graphically using the fourth order Runge-Kutta based shooting technique. Reddy et al [10] have studied unsteady flow of nonlinear radiative Powell-Eyring fluid past an inclined sheet with first order chemical reaction. Present investigations have an excellent agreement with the existed studies.

Keywords: Similarity Analysis, Unsteady Heat, Mass Transfer, Powell-Eyring Fluid Flow, Permeable Sheet.

Introduction

The non-Newtonian fluids flow has acquired a much adoration in the theory of fluid mechanics. Fluid, that do not obey Newton's law of motion are called non-Newtonian fluids. Some of researchers studied electrically conducting non-Newtonian fluid of the two dimensional magneto hydrodynamics boundary layer fluids. Mathematically, the Powell-Eyring model is more complex, and has advances of power law model, in case of power law model, it is deduced from kinematic theory of liquid rather than the empirical relation.

Heat transfer is the energy transfer due to a temperature difference in a mediator or between two or more mediums heat transfer can take place in one of the three process namely conduction, convection and radiation. Mass transfer is the displacement of the mass from one position to another. Prasad et al [9] expressed Powell-Eyring fluid flow characteristic with heat transfer in a non-isothermal stretching sheet. Na T. Y. et al [8] pioneered the application of Lie group method for determining the symmetries and invariant methods. Hayet et al [4] have studied the stagnation-point flow of the said fluid of melting heat transfer. Kheder et al [5] have pioneered the flow and heat transfer of Powell-Eyring fluid in an unsteady expanding sheet.

For the simultaneous occurrences of heat and mass transfer in between the fluxes, the driving potential is very complex, as the energy flux can be created not only by temperature gradient but also by composition gradient. The energy flux due to the concentration gradient is known as DuFour or diffusion-thermo effect. Khan et al [6] studied in detail Soret and DuFour effects for the double-diffusive convective flow of non-Newtonian Powell-Eyring fluid for a cone in a porous medium. D. Srinivasacharya et al [11] studied Soret and Dufour effects on mixed convection along a vertical wavy surface in a porous medium and discussed the effect of variables viscosity and the variable thermal conductivity on mixed convective heat and mass transfer past a vertical wavy surface embedded in a fluid saturated porous medium with Dufour and Soret effects. Also Brewster [2] has discussed the nature and properties of thermal radiative transfer.

Using Lie group analysis approach, Ma and Hui [7] studied similarity solutions of the two-dimensional unsteady boundary layer equation. Dholey and Gupta [3] extended this problem for moving the porous plate. Bluman and Kumei [1] have discussed in detail that Lie symmetric method is a symmetric mathematical procedure for determining symmetries of differential system.

In this paper, we investigate unsteady heat and mass transfer for non-Newtonian Powell-Eyring fluid flow over an expanding/shrinking sheet with suction/injection taking into consideration for the Soret and Dufour effects with thermal radiation. By taking the Lie symmetric technique the governing system of partial differential equation. The obtained differential equations can be solved numerically using shooting method with the help of MATLAB software. The present investigation has an excellent argument with the existed studies.

Mathematical Formulation

Consider the heat and mass transfer of two-dimensional unsteady boundary layer flow of a viscous incompressible non-Newtonian Powell-Eyring fluid along with an expanding/shrinking sheet with non-uniform suction, Dufour-Soret effects, and thermal radiation. The Geometrical and Physical sketch of the flow problem is given in Fig. 1.

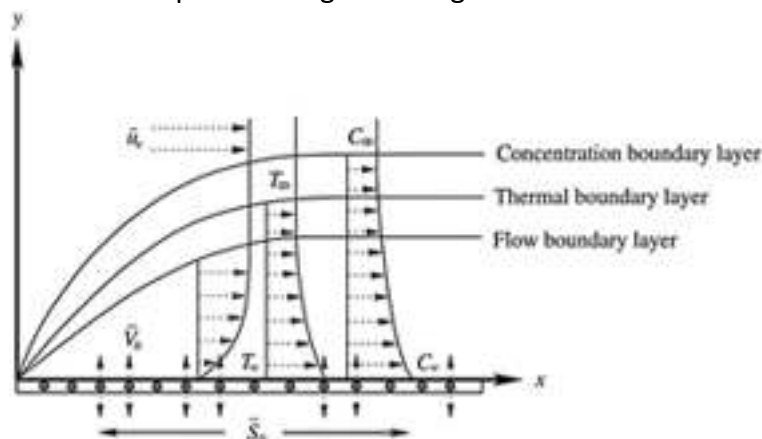


Figure 1

The shear-stress tensor for the Powell-Eyring fluid is given by

$$\tau_{xy} = \mu \frac{\partial \bar{u}}{\partial \bar{y}} + \frac{1}{\beta_1} \sinh^{-1} \left(\frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}} \right) \quad (1)$$

where μ is the coefficient of viscosity, β_1 and C_1 are material fluid parameters. The second order approximation of $\sinh^{-1} \left(\frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}} \right)$ function is given by

$$\sinh^{-1} \left(\frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}} \right) \cong \frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}} - \frac{1}{6} \left(\frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}} \right)^3, \left| \frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}} \right| \ll 1 \quad (2)$$

Under the usual boundary layer approximations, the governing equations of the mass, momentum, energy and concentration for the unsteady flow can be written as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (3)$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial \bar{u}_e}{\partial \bar{t}} + \bar{u}_e \frac{\partial \bar{u}}{\partial \bar{t}} + \left(\nu + \frac{1}{\rho \beta_1 C_1} \right) \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} - \frac{1}{2\rho \beta_1 C_1^3} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \quad (4)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho c_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho c_p} \frac{\partial \bar{q}_r}{\partial \bar{y}} + \frac{D_m k_1}{c_s c_p} \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \quad (5)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D_m \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{D_m k_1}{T_m} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \quad (6)$$

with boundary conditions,

$$\bar{u}(\bar{x}, \bar{y}, \bar{t}) = \bar{S}_0(\bar{x}, \bar{t}), \bar{v}(\bar{x}, \bar{y}, \bar{t}) = \bar{V}_0(\bar{x}, \bar{t}), \bar{T} = \bar{T}_w(\bar{x}, \bar{t}), \bar{C} = \bar{C}_w(\bar{x}, \bar{t}) \text{ at } \bar{y} = 0 \quad (7)$$

$$\bar{u}(\bar{x}, \bar{y}, \bar{t}) \rightarrow \bar{u}_\infty(\bar{x}, \bar{t}), \bar{T} \rightarrow T_\infty, \bar{C} \rightarrow C_\infty \text{ at } \bar{y} \rightarrow \infty \quad (8)$$

where $\bar{x}$ direction is along with the plate, $\bar{y}$ perpendicular to the plate, $\bar{t}$ is the time and $\bar{u}$ and $\bar{v}$ are, respectively, the velocity components in $\bar{x}$ and $\bar{y}$ directions, ν is the kinetic viscosity, $\bar{T}$ is the temperature, $\bar{C}$ is the concentration, κ is the thermal conductivity of the fluid, ρ is the constant fluid density, c_p the specific heat at constant pressure, $\bar{q}_r$ is the radiation heat-flux, D_m is the coefficient of mass diffusivity, k_t is the thermal diffusion ratio, c_s is the concentration susceptibility, T_m is the mean fluid pressure, $\bar{S}_0(\bar{x}, \bar{t})$ is the moving surface velocity, $\bar{V}_0(\bar{x}, \bar{t})$ is the suction or injection velocity of the surface, $\bar{S}_w(\bar{x}, \bar{t})$ is the surface temperature, $\bar{C}_w(\bar{x}, \bar{t})$ is the surface concentration, $\bar{u}_\infty(\bar{x}, \bar{t})$ is the free stream velocity outside the boundary layer and T_∞ and C_∞ are uniform ambient temperature and concentration of the flow respectively.

Symmetry Analysis

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (10)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial u_e}{\partial t} + u_e \frac{\partial u}{\partial x} + (1 + \varepsilon) \frac{\partial^2 u}{\partial y^2} - \varepsilon \delta \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} \quad (11)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho_r} \frac{\partial^2 T}{\partial y^2} + \frac{16}{3R} \frac{\partial^2 T}{\partial y^2} + Du \frac{\partial^2 c}{\partial y^2} \quad (12)$$

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{1}{S_c} \frac{\partial^2 y}{\partial y^2} + S_r \frac{\partial^2 t}{\partial y^2} \quad (13)$$

$$u(x, y, t) = S_0(x, t), v(x, y, t) = V_0(x, t), T = 1, C = 1 \text{ at } y = 0 \quad (14)$$

$$u(x, y, t) \rightarrow u_c(x, t), T \rightarrow 0, C \rightarrow 0 \text{ at } y \rightarrow \infty \quad (15)$$

Consider $u = \frac{\partial \psi}{\partial y}$; $v = -\frac{\partial \psi}{\partial x}$; and taking $\eta = yt^{-\frac{1}{2}}$; $\psi = xf(\eta)t^{-\frac{1}{2}}$; $u_e = xt^{-1}k$ (k is constant)

From equation (11), we obtain,

$$-f' - \frac{1}{2}\eta f'' + (f')^2 - ff'' = (-k + k^2) + (1 + \varepsilon)f''' - \varepsilon\delta \frac{x^3}{t^3}(f'')^2 f''' \quad (16)$$

Also consider

$$\eta = yt^{-\frac{1}{2}}, C = xt^{-2}\theta(\eta), T = xt^{-2}\phi(\eta) \quad (17)$$

From (12), we obtain,

$$-2\theta - \frac{1}{2}\eta\theta' + f'\theta - f\theta' = \frac{1}{S_c}\theta'' + S_r\phi'' \quad (18)$$

And from (13), we obtain,

$$-2\phi - \frac{1}{2}\eta\phi' + f'\phi - f\phi' = \left(\frac{1}{P_r} + \frac{16}{3R}\right)\phi'' + D_u\theta'' \quad (19)$$

These are ordinary differential equations under the such conditions obtained.

The equation (16) can be solved using usual methods, since it is in one dependent variable. So that, the equation becomes

$$-f' - \frac{1}{2}\eta f'' + (f')^2 - ff'' = (-k + k^2) + (1 + \varepsilon)f''' - (f'')^2 f''' \quad (20)$$

$$-\frac{\eta}{2}\phi' - f\phi' = \frac{1}{S_c}\phi'' + S_r\theta'' \quad (21)$$

$$f' = a, f = b, \theta = 1, \phi = 1 \text{ at } \eta = 0 \quad (22)$$

$$f' \rightarrow c, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (23)$$

Numerical Solution

The above non-linear differential equations (20), (21) along with boundary conditions (22), (23) form a two-point boundary value problem and are solved using shooting method, by converting it into an initial value problem. In this method, we have chosen a suitable finite value of $\eta \rightarrow \infty$, say η_∞ . We set the following first order systems

$$f' = z, z' = p, \phi' = r, \theta' = s \quad (24)$$

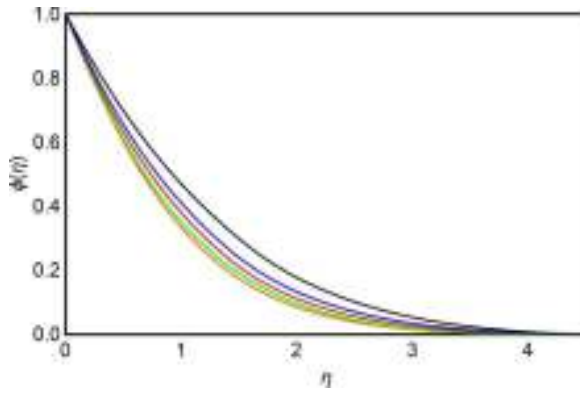
$$p' = \frac{1}{1+\varepsilon-\varepsilon p^2} \left\{ -\left(\frac{1}{2}\eta + f\right)p + (z^2 - z) + (k - k^2) \right\} \quad (25)$$

$$r' = \frac{(-2+z)\theta - \left(\frac{1}{2}\eta + f\right)s + \left\{(2+z)\phi + \left(\frac{1}{2}\eta + f\right)r\right\}}{S_c Du \left\{S_r - \left(\frac{1}{P_r} + \frac{16}{3R}\right)\right\}} \quad (26)$$

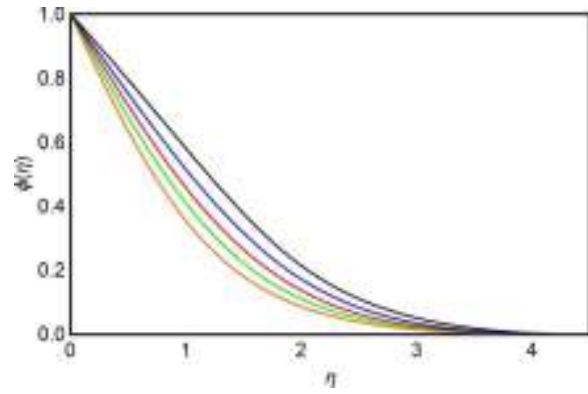
$$s' = \frac{(2-z)\phi + \left(\frac{1}{2}\eta + f\right)r + \frac{1}{S_c} \left(\frac{1}{P_r} + \frac{16}{3R}\right) \left\{(-2+z)\theta - \left(\frac{1}{2}\eta + f\right)s\right\}}{Du - \frac{1}{S_c S_r} \left(\frac{1}{P_r} + \frac{16}{3R}\right)} \quad (27)$$

$$\text{with } f(0) = b, z(0) = a, \theta(0) = 1, \phi(0) = 1. \quad (28)$$

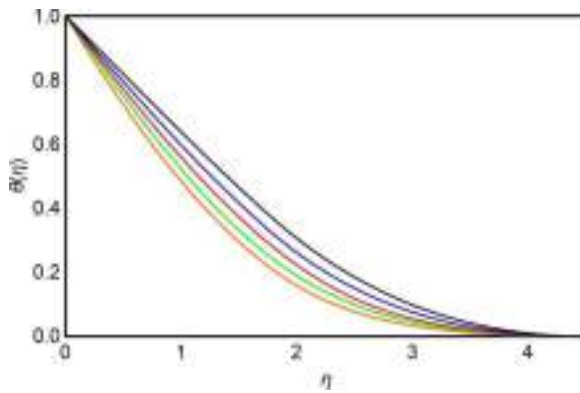
To solve the initial value problem we have to consider values for $f''(0)$, $\theta'(0)$, $\phi'(0)$ and fourth order Runge-Kutta method to obtain the solution. The numerical solutions are illustrated graphically in the form of non-dimensional velocity, temperature, concentration profile. We consider the values of the governing parameter as $\varepsilon = 0.2$, $\delta = 0.6$, $P_r = 7.0$, $R = 2.0$, $D_u = 0.4$, $S_c = 0.6$, $S_r = 0.5$, $c = 0.4$.



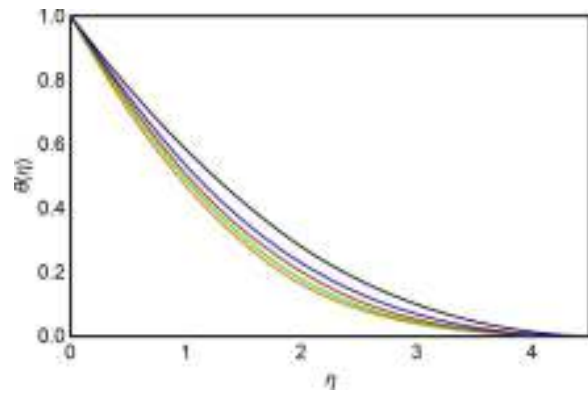
Graph 1
Concentration profile for a
 $a = -0.6, -0.3, 0, 0.3, 0.6$ and $b = 1.0$



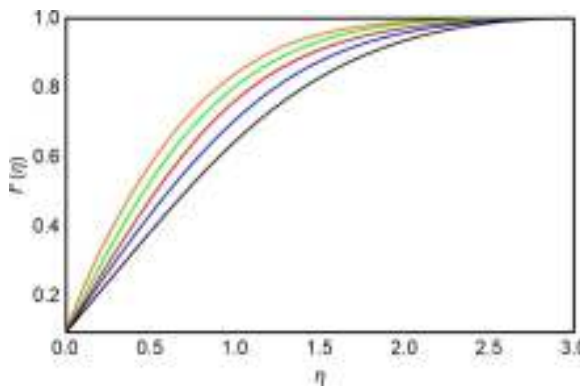
Graph 2
Concentration profile for b
 $b = -0.6, -0.3, 0, 0.3, 0.6$ and $a = 0.1$



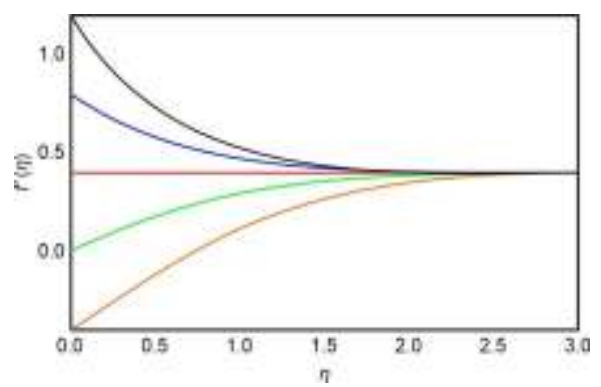
Graph 3
Temperature profile for a
 $a = -0.6, -0.3, 0, 0.3, 0.6$ and $b = 1.0$



Graph 4
Temperature profile for b
 $b = -0.6, -0.3, 0, 0.3, 0.6$ and $a = 0.1$



Graph 5
Velocity profile for b
 $b = -0.6, -0.3, 0, 0.3, 0.6$ and $a = 0.1$



Graph 6
Velocity profile for a
 $a = -0.4, 0, 0.4, 0.8, 1.2$ and $b = 1.0$

Conclusion

The equations are solved using numerical techniques and the results are represented graphically. Graph (1), (3) and (6) present effects of expanding/shrinking parameter a on concentration, temperature and velocity respectively. It is clear that $a > 0$ corresponds to expanding sheet, $a < 0$ corresponds to shrinking sheet. The suction/injection parameter b of fluid mass has impact on concentration, temperature and velocity profiles which is graphically illustrated in graph (2), (4) and (5) respectively. In a similar way the graphical study can be done using the Dufour number Du , Soret number S_r and Prandtl number P_r which is not included in this paper.

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